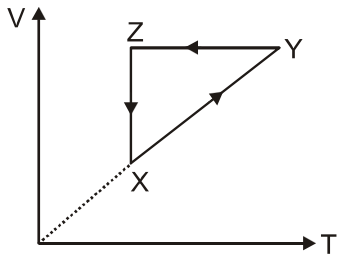
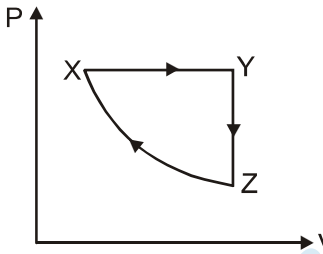
Memory Based_JEE Main Online Test_08-01-20_Morning

Physics

1. Given V-T diagram, draw the corresponding P-V diagram.

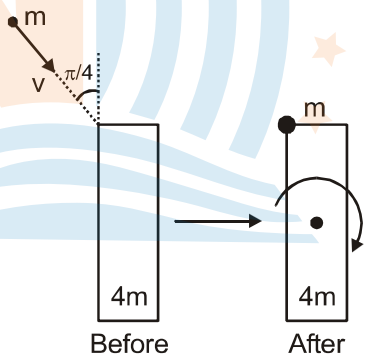


Sol.



2. A uniform rod of mass 4m is pivoted at centre. A particle of mass m strikes with velocity v an the one end of rod at an angle $\frac{\pi}{4}$ and it sticks with it after collision. Then what will be the angular velocity after collision.

Sol. $\left(\frac{\ell}{2}\right)m\times v\sin 45^\circ=\left[\frac{4m\ell^2}{12}+m\left(\frac{\ell}{2}\right)^2\right]\omega$



$$\frac{v}{2\sqrt{2}}=\ell\left[\frac{1}{3}+\frac{1}{4}\right]\omega$$

$$\frac{v}{2\sqrt{2}}=\ell\left[\frac{7}{12}\right]\omega$$

$$\omega=\frac{3\sqrt{2}v}{7\ell}$$

3. Relative electric permittivity and relative magnetic permeability of a given medium are 3 and 4/3 respectively. Then find critical angle for given medium.

Sol. Here, $v=\frac{C}{\sqrt{\mu_r\varepsilon_r}}=\frac{C}{\sqrt{3\times\frac{4}{3}}}=\frac{C}{2}$

$\therefore\ \mu=\frac{C}{V}=\frac{C}{C/2}=2$

$\therefore$ Critical angle = $\sin^{-1}\left(\frac{1}{\mu}\right)$ = 30°.

4. Two capacitors are in parallel with effective capacitance 10 μF. The energy stored in the C_1 when it is corrected to 1V vattery is twice the energy stared in the C_2 when it is connected to 1V battery. Find the value of effective capacitance when they are in series.

Sol. $C_1 + C_2 = 10\ \mu F$ .....(1)

$$\frac{1}{2}C_1V^2 = 4 \times \frac{1}{2}C_2V^2$$

$C_1 = 4C_2$ .....(2)

From (1) & (2)

$5C_2 = 10\ \mu F$

$C_2 = 2\ \mu F$

$C_1 = 8\ \mu F$

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{16}{10} = 1.6\ \mu F$$

5. Two particles A and B of same mass 0.1 kg moving with velocity $V_A = 3\hat{i}$ and $V_B = 5\hat{j}$ respectively, collide elastically. After collision A is moving with velocity $\vec{V}_A = 4(\hat{i} + \hat{j})$. The energy of B is given as $\frac{x}{10}J$. Find the value of x.

Sol. Using conservation of momentum

$$(0.1)(3\hat{i} + 5\hat{j}) = (0.1)\,(4(\hat{i} + \hat{j}) + V_B)$$

$$3\hat{i} + 5\hat{j} = 4\hat{i} + 4\hat{j} + V_B$$

$$-\hat{i} - \hat{j} = V_B$$

K.E. of B = $\frac{1}{2}mV_B^2$

$$= \frac{1}{2} \times \frac{1}{10} \times 2$$

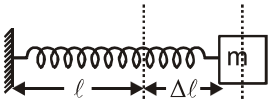
$$= \frac{1}{10}J$$

Value of x = 1.

6. A block of mass m kept in gravity free space attached to a spring of spring constant k is rotated with angular frequency ω, then find extension in spring.

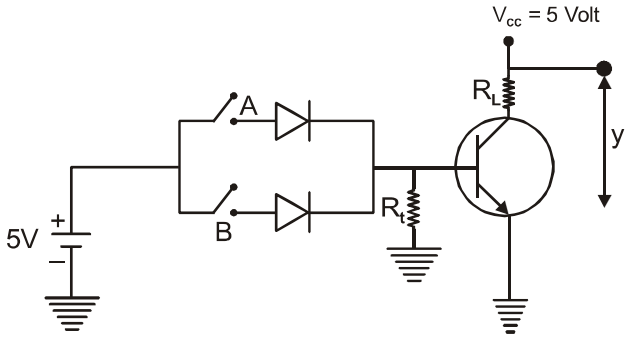
Sol. $m\omega^2(\ell + \Delta\ell) = k\Delta\ell$

$m\omega^2 = (k - m\omega^2)\,\Delta\ell$



$$\Delta\ell = \frac{m\omega^2\ell}{k - m\omega^2}$$

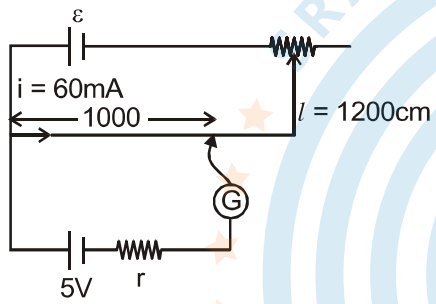
7. Input given are A and B, then the value of output 'y' will be



Sol. $y = \overline{(A + B)}$

8. A potentiometer circuit, with wire of length 1200 cm and current passing throguh is 60 mA. The second circuit contains 5V battery with 20Ω internal resistance. Null point is found at 1000 cm. Find the value of resistance of potentiometer wire.

Sol.



$5 = i \times R_1 \Rightarrow R_1 = \frac{5}{i}$

$R \propto k \Rightarrow \frac{R}{R_1} = \frac{1200}{1000} = \frac{6}{5}$

$R = \frac{6}{5} \times \frac{5}{i}\ \Omega$

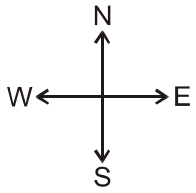
$R = \frac{6}{5} \times \frac{5}{60 \times 10^{-3}} = 100\ \Omega$

9. A proton is moving in a magnetic field with accleration 10^{12} m/sec^2 from south to north direction. what is the magnetic field if the kinetic energy of proton is 1 Mev (Rest mass of proton = 1.67×10^{-27} kg)

Sol. $m\vec{a} = q(\vec{V} \times \vec{B})$

$m\vec{a} = q(V\hat{j} \times Bi)$

$\vec{a} = \frac{qVB}{m}\hat{K}$



$a = \left(\frac{qB}{m}\sqrt{\frac{2K}{m}}\right)$

$a = qB\ \frac{\sqrt{2k}}{m^{3/2}}$

$B = \frac{am^{3/2}}{\sqrt{2kq}}$

10. A uniform cylinder at 0ºC of lenght 1m is partially submerged in water, having 20 cm length outside of water. Now the tempertaure. of water is increased by 1ºC so that outside length is also increased by 1 cm, what is the ratio of density of water at 4ºC and 0ºC.

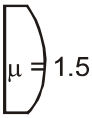
Sol. $R.D = \frac{\rho}{\sigma_{0^\circ C}} = \frac{20}{100}$

$R.D' = \frac{\rho}{\sigma_{4^\circ C}} = \frac{21}{100}$

$\frac{\sigma_{4^\circ C}}{\sigma_{0^\circ C}} = \frac{20}{21}$

11. Plano Convex lens of radius of curvature R = 30 cm and refractive index = 1.5 then find focal length of lens.

Sol. R = 30 cm



$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-30}\right)$

$\frac{1}{f} = \left(\frac{1}{2}\right)\left(\frac{1}{30}\right)$

f = 60 cm

12. Consider two particle A and B moving in a straight line is given by $x = 8t - 3t^2 + 10$ and $y = 5 - 8t^3$. If velocity of A w.r.t B at t = 1 sec is V. then find $\sqrt{V}$.

Sol. $V_B = \frac{dy}{dt} \Rightarrow V_B = \frac{d\left(5-8t^3\right)}{dt}$

$V_B = 0 - 24t^2$

$V_B = -24t^2$

$V_A = \frac{dx}{dt} \Rightarrow V_A = \frac{d\left(8t-3t^2+10\right)}{dt}$

$V_A = 8 - 6t$

$V_A = 8 - 6(1)$

$V_A = 2$

$\left|\vec{V}\right| = \sqrt{V_A{}^2 + V_B{}^2}$

$\left|\vec{V}\right| = \sqrt{2^2 + (-24)^2}$

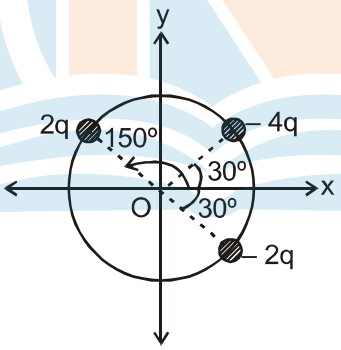
$\left|\vec{V}\right| = \sqrt{580}$

13. Gaus Law $\oint \vec{E} \cdot d\vec{A} = \frac{\sum Q_{in}}{\varepsilon_0}$ is appliceble in

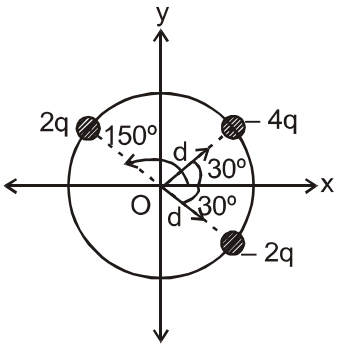
(A) In uniform Electric field only
(B) On equipotential surface.
(C) In uniform electric field & equipotential surface
(D*) On any Gaussin surface.

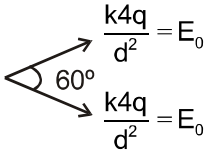
Ans. **(D)**

14. Find x component of electric field at origin.



Sol.





$E_R = \sqrt{3}E_0$; $E_R = \frac{\sqrt{3}K4q}{d^2}$

15. A sphere of non uniform density as $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$. Find the minimum density of liquid so that it will float.

Sol. $d_{liquid} > d_{body}$ (Condition of floating)

For minimum density

density of liquid = density of body

$= mg = \sigma V_i g$

$$m=\int_0^R \rho dv$$

$$m=\int_0^R \rho_0\left(1-\frac{r^2}{R^2}\right)4\pi r^2 dr$$

$$m=4\pi\rho\int_0^R\left(r^2-\frac{r^4}{R^2}\right)dr$$

$$m=4\pi\rho_0\left[\frac{R^3}{3}-\frac{R^5}{5}\times\frac{1}{R^2}\right]$$

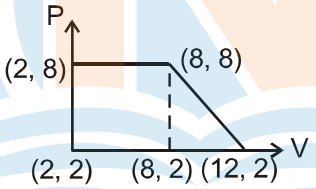
$$m=4\pi\rho_0 R^3\left[\frac{2}{15}\right]$$

$mg = \sigma Vg$

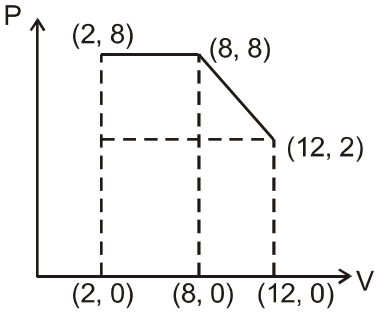
$$\frac{8}{15}\pi\rho_0 R^3 g=\sigma\frac{4}{3}\pi R^3 g$$

$$\sigma=\frac{2}{5}\rho_0$$

16. Find work done by the gas during the given process?



Sol.



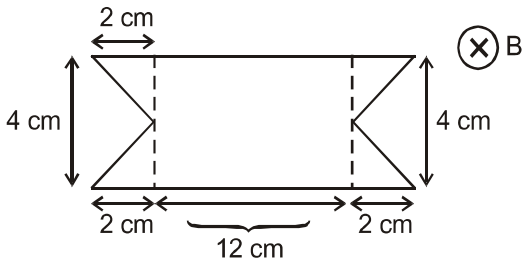
Work done by gas is area under the curve towards v-axis in P-V curve.

$$\Delta w = 6\times 8 + 4\times 2 + \frac{1}{2}\times 4\times 6$$

$= 48 + 8 + 12$

$= 68$ J

17. In the following diagram magentic field changes from 1000 G to 500 G in 5 sec. Find induced E.M.F in the diagram.



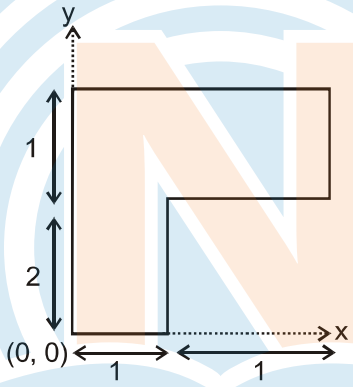
Sol. $E = -\frac{d\phi}{dt}$ $\{\phi = BA\}$

$E = -\frac{AdB}{dt}$

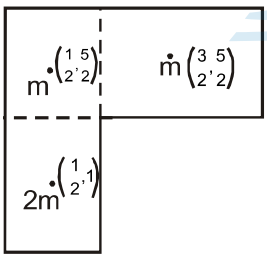
$A = 56 \times 10^{-4}$

$E = \frac{-AdB}{dt} = -56\times10^{-4}\times100\times10^{-4} = -56\times10^{-6}$

18. What is the Co-ordinate of centre of mass of the uniform sheet



Sol.



$$X_{CM} = \frac{\left(\frac{1}{2}\right)(2m)+\left(\frac{1}{2}\right)m+m\left(\frac{3}{2}\right)}{4m} = \frac{1+\frac{1}{2}+\frac{3}{2}}{4}$$

$$Y_{CM} = \frac{(1)(2m)+\left(\frac{5}{2}\right)(m)+\left(\frac{5}{2}\right)m}{4m} = \frac{7}{4}$$

19. Photon of energy E = 4 eV incident on metal A than maximum kinetic energy of e^- is T_A and photon of energy E = 4.5 eV incident an metal B then max. K.E. of e^- is T_B . If De broglie wavelength of e^- from metal A is λ_A and from metal B is λ_B . Given $\lambda_B = 2\lambda_A$ and $T_B = T_A - 1.5$ eV then find work function of metal B.

Sol. $\lambda_B = 2\lambda_A$

$$\left(\frac{\lambda_A}{\lambda_B} = \frac{1}{2}\right) \Rightarrow \left(\lambda = \frac{h}{\sqrt{2mE}}\right)$$

$$\frac{T_B}{T_A} = \frac{1}{4}$$

$T_A = 4T_B$

given $T_B = T_A - 1.5$ eV.

$3T_B = 1.5$ eV

$T_B = 0.5$ eV

$T_B = 4.5 - \phi_B$

$\phi_B = 4$eV

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Chemistry

1. Number of S–O bonds in S–O bonds in $S_2O_8^{2-}$ ion.
Number of S–S bonds in Rhombic S_8 molecule.

Sol. **$S_2O_8^{2-} \rightarrow 8$ S–O bonds**
Rhombic $S_8 \rightarrow 8$ S–S bonds.

2. The complex that can show fac-mer isomerism.
Complex of $[Ma_4b_2]$ $[Ma_2b_2]$ $[Ma_3b_3]$ $[Ma_2(BB)_2]$ here given.

Sol. **$[Ma_3b_3]$ type complex will exibit fac-mer isomerism.**

3. In ethyl-acetate molecule, the type of force existing in it.

Sol. **Dipole-Dipole and Vanderwaal's forces of attraction.**

4. The correct order of 1st ionisation energy for Na, Mg, Al, Si

Sol. **1st I.E. : Na < Al < Mg < Si**

5. Which of the following is a greenhouse gas.

Sol. **CO_2, CH_4, CFC, water vapour, Ozone and Nitrous oxide**

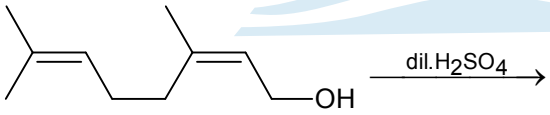
6. The element with maximum value of 3rd ionisation energy.
(1) Mn (2) Fe (3) Co (4) Ni

Sol. **Mn has maximum 3rd I.E.**

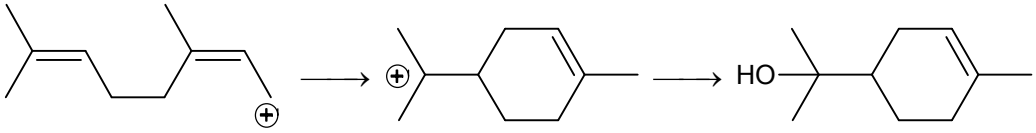
7. Heating of gypsum at 393 K results in :
(1) Dead Burnt plaster.
(2) Anhydrous calcium sulphate
(3) $CaSO_4 \cdot 5H_2O$
(4) $CaSO_4 \cdot 0.5H_2O$

Sol. **Plaster of paris $CaSO_4 \cdot \frac{1}{2}H_2O$ is formed.**

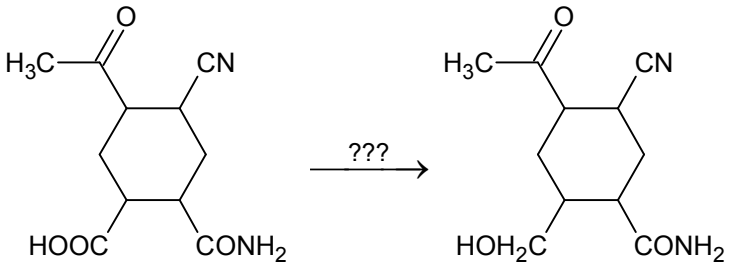
8.



Sol.



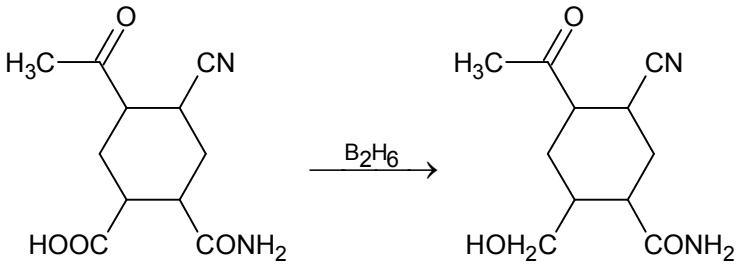
9.



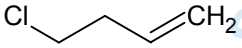
Which reagent can be used for above conversion :

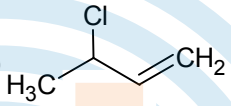
(1) $NaBH_4$ (2) $LiAlH_4$ (3) H_2/Pd (4) B_2H_6

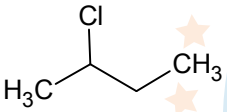
Sol.



10. Find correct rate of dehydrohalogenation / E_1

(a) 

(b) 

(c) 

(d) 

Sol. b > a > c > d

11. Iso-hexane and 3-methyl pentane are two compounds with B.P. of 60°C and 63°C. They can be seperated by and which will come out first

(a) fractional distillation isohexane > 3-methyl pentane

(b) Simple distillation 3-methyl pentane > isohexane

(c) Fractional distillation 3-methyl pentane > isohexane

(d) Simple distillation isohexane > 3-methyl pentane

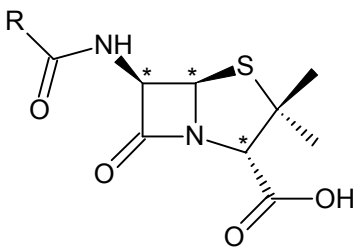
Find the correct statement from above :

Sol. (a) fractional distillation isohexane > 3-methyl pentane

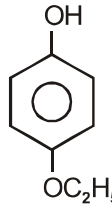
12. Number of chiral center in pencilline ?

Ans. **3**

Sol.



13. C—O Bond length order will be in Methanol, Phenol, P–ethoxy phenol

Sol. (a) $CH_3—OH$ (Methanol) > (c)  (P- ethoxy phenol) > (b) $Ph—OH$ (Phenol)

14. **Incorrect** statement :

(1) Glucose gives schiff reagent test

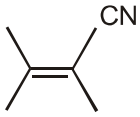
(2) Glucose gives hydroxylamine test

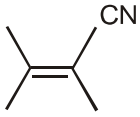
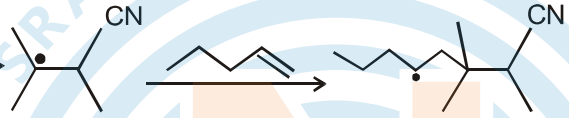
(3) Glucose is found to exist in two different crystalline forms which are named as α- glucose and β-glucose

(4) The penta acetate of glucose does not react with hydroxylamine

Ans. **(1)**

Sol. **Glucose gives schiff reagent test negative**

15.  Peroxide → (X) ; X +  → (Y) Identify X and Y

Sol.  Peroxide → 

16. For the given reaction

$2H_2O(\ell) \longrightarrow O_2(g) + 4H^+(aq) + 4e^-$ $E^\circ = -1.23$ V

Find electrode potential (V) at pH = 5 and P_{O_2} = 1 bar at 25°C (R = 8.314 J/K mol)

Ans. **–0.93**

Sol. $E = E^\circ - \frac{0.059}{4}(10^{-5})^4 = -1.23 + 0.059 \times 5 = -0.935$ V

17. At a given specific temperature (T) due to enzymatic reaction, rate constant $k_2 = 10^6 k_1$.

Find change in activation energy :

Ans. 6×2.303 RT

Sol. $\ln\frac{k_2}{k_1} = \frac{\Delta E_a}{RT}$; $2.303 \log 10^6 = \frac{\Delta E_a}{RT}$; 2.303×6 RT $= \Delta E_a$

18. For Balmer series in emission spectrum of H–atom $\bar{\nu} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$, following statements are given :

(a) Spectral lines converge as λ decreases.

(b) $n_1 = 2$

(c) λ_{max} if $n_2 = 3$

(d) Ionisation energy of H–atom can be calculated by using the $\bar{\nu}$ of the equation.

Select correct statement

Sol. Statement b, c and d are correct.

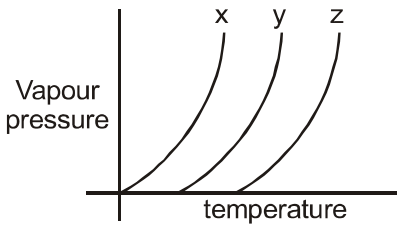
19. Find the volume (ml) of 0.125 M $AgNO_3$(aq) required for complete precipitation of AgCl (s) when treated with 0.3g $[Co(NH_3)_6]Cl_3$ (GMM= 266.5).

Ans. **27.02**

Sol. $3\,AgNO_3 + [Co(NH_3)_6]Cl_3 \longrightarrow [Co(NH_3)_6](NO_3)_3 + AgCl\ (ppt)$

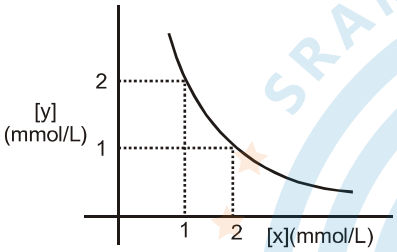
$$\frac{0.125 \times V}{3 \times 1000} = \frac{0.3}{266.5} = 27.02 \text{ ml}$$

20. A plot between vapour pressure and temperature is given for three volatile liquids x, y and z. Find correct order of intermolecular attraction forces.



Ans. z > y > x order of attraction forces.

21.



Select the correct option for given sparingly soluble salt(s) :

(1) xy_2 $K_{sp} = 4 \times 10^{-6}\ M^3$

(2) x_2y $K_{sp} = 9 \times 10^{-6}\ M^3$

(3) xy $K_{sp} = 2 \times 10^{-6}\ M^2$

(4) xy_2 $K_{sp} = 1 \times 10^{-6}\ M^3$

Ans. **(3)**

Sol. $K_{sp}(xy) = [x]\,[y] = 2 \times 10^{-6}\ M^2$

22. Find the mass of ferrous sulphate heptahydrate ($FeSO_4 \cdot 7H_2O$, GMM = 278) present in 100 kg wheat sample having 10 ppm of iron (GMM = 56)

Ans. **4.96 gm**

Sol. $$\text{ppm} = \frac{W_{Fe}}{W_{sample}} \times 10^6$$

$$10 = \frac{W_{Fe}}{100 \times 10^3} \times 10^6$$

W_{Fe} = 1 gm

Weight of $FeSO_4 \cdot 7H_2O = \frac{278}{56} \times 1 = 4.96$ gm

Memory Based_JEE Main Online Test_08-01-20_Morning

MATHEMATICS

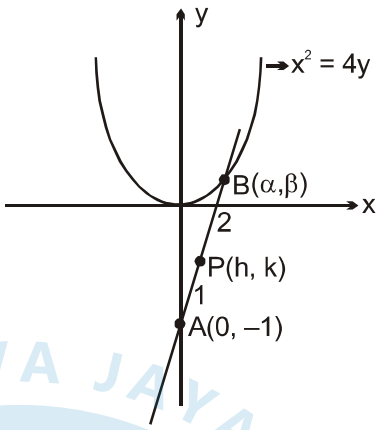
1. A line passing through A(0, –1) intersect $x^2 = 4y$ at a point B. Find locus of the point which divides line segment AB in 1 : 2

Ans. **8**

Sol. Let (α, β) lies on $x^2 = 4y$

$\because \alpha^2 = 4\beta$ .....(1)

Let p(h, k) is the point such that



$$\frac{AP}{BP} = \frac{1}{2}$$

$$h = \frac{\alpha}{3}, k = \frac{\beta - 2}{3}$$

$\alpha = 3h$, $\beta = 3k + 2$

Put in (1)

$(3h)^2 = 4(3k + 2)$

locus is $9x^2 = 4(3y + 2)$

2. If maximum value of ${}^{19}C_p$, ${}^{20}C_q$, ${}^{21}C_r$ are a, b, c respectively, then which of the following is correct.

Ans. $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$

Sol. $a = {}^{19}C_9$

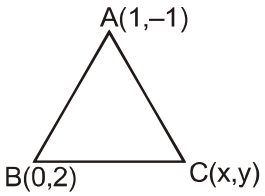
$$b = {}^{20}C_{10} = \frac{20}{10}.{}^{19}C_9 = 2a$$

$$c = {}^{21}C_{11} = \frac{21}{11}.{}^{20}C_{10} = \frac{21}{11}b = \frac{42}{11}a$$

$$\therefore \quad \frac{42a}{11} = \frac{21b}{11} = c \Rightarrow \frac{a}{11} = \frac{b}{22} = \frac{c}{42}$$

3. If area of triangle formed by (1, –1), (0, 2) & (x, y) is 5 then find value of λ such that $3x + y - 4\lambda = 0$.

Sol. Area = 5



$$\frac{1}{2}\begin{vmatrix} 1 & -1 \\ 0 & 2 \\ x & y \\ 1 & -1 \end{vmatrix} = \pm 5$$

$$\frac{1}{2}(2 - x - 2x - y) = \pm 5$$

$2 - 3x - y = \pm 10$

$3x + y \pm 10 - 2 = 0$

$3x + y + 8 = 0$ or $3x + y - 12 = 0$

$-4\lambda = 8, -4\lambda = -12$

$\lambda = -2$ $\lambda = 3$

4. If for $f(x) = \ell n\left(\frac{x^2+\alpha}{7x}\right)$, Rolles theorem is applicable in [3, 4] for $c \in (3, 4)$, then f''(c) is equal to

Ans. $\frac{1}{144}$

Sol. $f(3) = f(4) \Rightarrow \frac{9+\alpha}{21} = \frac{16+\alpha}{28} \Rightarrow 36 + 4\alpha = 48 + 3\alpha \Rightarrow \alpha = 12$

$\therefore f(x) = \ell n\left(\frac{x^2+12}{7x}\right) = \ell n(x^2 + 12) - \ell n(7x)$

$\Rightarrow f'(x) = \frac{2x}{x^2+12} - \frac{1}{x}$

$\Rightarrow f''(x) = \frac{2(x^2+12)-4x^2}{(x^2+12)^2} + \frac{1}{x^2}$

$f'(c) = 0 \Rightarrow c^2 = 12$

$\therefore f''(c) = \frac{1}{144}$

5. Find the value of $\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$

Ans. $\frac{1}{e^2}$

Sol. $\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}} = e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left[\frac{3x^2+2}{7x^2+2}-1\right]} = e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left(\frac{-4}{7x^2+2}\right)} = e^{-2} = \frac{1}{e^2}$

6. The inverse of function $f(x) = \frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$; $x \in (-1, 1)$ is

Ans. $f^{-1}(x) = \frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

Sol. $f(x) = y \Rightarrow x = f^{-1}(y)$

$\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}} = y \Rightarrow (y + 1)\, 8^{-2x} = (1 - y)\, 8^{2x}$

$\Rightarrow 8^{4x} = \frac{1+y}{1-y} \Rightarrow x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right) = f^{-1}(y)$

7. If A and B are independent events such that P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{6}$ then which of the following is correct.

(1) $P\left(\frac{A}{B}\right)=\frac{2}{3}$ (2) $P\left(\frac{A}{A'\cap B'}\right)=\frac{1}{3}$ (*C) $P\left(\frac{A'}{B'}\right)=\frac{2}{3}$ (D) None of these

Ans. (C)

Sol. $\therefore \quad P\left(\frac{A'}{B'}\right)=P(A')=1-P(A)=\frac{2}{3}$

8. If $f(x) = [\sin(\tan^{-1}x) + \sin(\cot^{-1}x)]^2 - 1$ and $y\left(\sqrt{3}\right)=\frac{\pi}{6}$ $\frac{dy}{dx}$ = F′(x), then $y(-\sqrt{3})$ is equal to

Sol. $f(x) = [\sin(\tan^{-1}x) + \sin(\cot^{-1}x)]^2 - 1$

Put $\tan^{-1}x = \theta$

$f(x) = (\sin\theta + \cos\theta)^2 - 1$

$f(x) = \sin[2\tan^{-1}x]$

Given

$\frac{dy}{dx}=f'(x)$

$y = f(x) + C$

$y = \sin(2\tan^{-1}x) + C$

$y(\sqrt{3})=\frac{\pi}{6}$

on solving

$C=\left(\frac{\pi}{6}-\frac{\sqrt{3}}{2}\right)$

$y(-\sqrt{3}) \rightarrow ?$

$\boxed{y=\frac{\pi}{6}-\sqrt{3}}$ **Ans.**

9. How many square matrix A of order 3 can be formed whose elements are –1, 0, 1 and trace of AA^T is 3.

Ans. **672**

Sol. $AA^T=\begin{bmatrix}a & b & c\\ d & e & f\\ p & q & r\end{bmatrix}\begin{bmatrix}a & d & p\\ b & e & q\\ c & f & r\end{bmatrix}$

$\therefore$ Trace of $AA^T = a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + p^2 + q^2 + r^2 = 3$

$\therefore$ Number of ways = ${}^9C_3 \times 2^3 = 672$

10. If $\sqrt{1-x^2}\,\frac{dy}{dx}+\sqrt{1-y^2}=0$ and $y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$, then the value of $y\left(\frac{1}{\sqrt{2}}\right)$ is equal to

Ans. $\frac{1}{\sqrt{2}}$

Sol. $\frac{dy}{\sqrt{1-y^2}}+\frac{dx}{\sqrt{1-x^2}}=0 \Rightarrow \sin^{-1}y + \sin^{-1}x = C$

$\because \quad y\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2} \Rightarrow C=\frac{\pi}{2} \Rightarrow \sin^{-1}y+\sin^{-1}x=\frac{\pi}{2}$

$\Rightarrow \quad y\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{\sqrt{2}}$

11. If $\int\frac{\cos x}{\sin^3 x(1+\sin^6 x)^{2/3}}dx = f(x)\,(1+\sin^6 x)^{1/\lambda}+C$, then the value of $\lambda\cdot f\left(\frac{\pi}{3}\right)$ is

Ans. –2

Sol. For sinx = t

$$I=\int\frac{dt}{t^3\left(1+t^6\right)^{\frac{2}{3}}}=\int\frac{dt}{t^7\left(\frac{1}{t^6}+1\right)^{\frac{2}{3}}}=-\frac{1}{2}\left(1+\frac{1}{t^6}\right)^{\frac{1}{3}}$$

$$I=-\frac{1}{2t^2}\left(1+t^6\right)^{\frac{1}{3}}=\frac{-1}{2\sin^2 x}\left(1+\sin^6 x\right)^{\frac{1}{3}}+C$$

$\therefore \quad f(x)=-\frac{1}{2\sin^2 x}$ and $\lambda = 3$

$\Rightarrow \quad \lambda f\left(\frac{\pi}{3}\right)=3\times\frac{-1}{\frac{3}{2}}=-2$

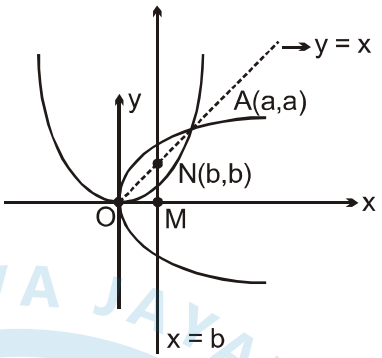
12. If $c_1 : y^2 = ax$, $c_2 : x^2 = ay$. A line $x = b$ divides the area bounded by c_1 & c_2 in to two equal parts. Also the area of triangle bounded by $x = b$, $y = x$ & x axis is $\frac{1}{2}$, then which is true.

(1) $a^6 - a^3 + 4 = 0$ (2) $a^6 - 6a^3 + 5 = 0$ (3) $a^6 - 10a^3 + 4 = 0$ (*4) $a^6 - 12a^3 + 4 = 0$

Sol. ar $\Delta OMN = \frac{1}{2}$

$$\frac{1}{2}b.b = \frac{1}{2}$$

$$\boxed{b = 1} \qquad [\because\ b > 0]$$



Also given

$$\int_0^1 \left(\sqrt{ax} - \frac{x^2}{a}\right)dx = \int_1^a \left(\sqrt{ax} - \frac{x^2}{a}\right)dx$$

$$\frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{2}{3}\sqrt{a}(a\sqrt{a} - 1) - \frac{(a^3 - 1)}{3a}$$

$\Rightarrow$ $a^3 + 2 = 4a^{3/2}$

$\Rightarrow$ squaring we get

$a^6 - 12a^3 + 4 = 0$

13. Which of the following is tautology.

(1) $(p \wedge (p \to q)) \to q$ (2) $q \to p \wedge (p \to q)$

(3) $p \vee (p \wedge q)$ (4) $p \wedge (p \vee q)$

Ans. **(1)**

Sol.

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

14. Let $f(x) = [\sin(\tan^{-1}x) + \sin(\cot^{-1}x)]^2 - 1$ where $|x| > 1$ and $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1}f(x))$. If $y(\sqrt{3}) = \frac{\pi}{6}$ then $y\left(-\sqrt{3}\right) =$

(1) $\frac{5\pi}{6}$ (*2) $\frac{-\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{2\pi}{3}$

Ans. **(2)**

Sol. $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}f(x)\right) \Rightarrow y = \frac{1}{2}\sin^{-1}(f(x)) + C$

$f(x) = [\sin(\tan^{-1}x) + \sin(\cot^{-1}x)]^2 - 1$

Let $\tan^{-1}x = \theta \Rightarrow \cot^{-1}x = \frac{\pi}{2} - \theta$

$f(x) = \sin 2\theta$

we get

$y = \frac{1}{2}\sin^{-1}\left(\sin\left(2\tan^{-1}x\right)\right) + C$

At $x = \sqrt{3}, y = \frac{\pi}{6}$

$\Rightarrow \boxed{C = 0}$

At $x = -\sqrt{3}$

$y = \sin^{-1}\left(\sin\left(2\tan^{-1}x\right)\right)$

$y = \sin^{-1}\left(\sin\left(2\tan^{-1}(-\sqrt{3})\right)\right)$

$y = -\frac{\pi}{6}$ **Ans.**

Subjective :

15. If $(3^x + 3^{-x})$, $f(x)$, $(2^{1+x} + 2^{1-x})$ are in A.P., then find minimum value of $f(x)$.

Ans. 3

Sol. $f(x) = \frac{1}{2}[3^x + 3^{-x} + 2^{1+x} + 2^{1-x}]$

$f(x) = \frac{1}{2}[3^x + 3^{-x} + 2(2^x + 2^{-x})] \geq 3, \forall x \in R$

16. Find $\sum_{k=1}^{20}(1 + 2 + 3..... + k)$

Sol. $\sum_{k=1}^{20}\frac{k(K+1)}{2} = \frac{1}{2}\left(\sum_{k=1}^{20}k^2 + \sum_{k=1}^{20}k\right) = \frac{1}{2}\left(\frac{20\times21\times41}{6} + \frac{20\times21}{2}\right) = 1540$

17. If 3 white, 4 black and 5 red balls are given. Find number of ways of selecting 4 bolls out of which atmost 3 are red.

Ans. **490**

Sol. Total – (all red) = $^{12}C_4 - {}^5C_4 = 495 - 5 = 490$

18. If normal at point P on curve $y^2 - 3x^2 + y + 10 = 0$ passes through the point $\left(0, \frac{3}{2}\right)$, then slope of tangent at point P is n. Then the value of |n| is

Ans. 4

Sol. Given : $y^2 + y - 3x^2 + 10 = 0$ .....(1)

Differentiate with respect to x

$2yy' + y' - 6x = 0$

$$y' = \frac{6x}{2y+1}$$

So slope of tangent at (x_1, y_1) is

$$\frac{dy}{dx} = \frac{6x_1}{2y_1+1} \quad \text{.....(2)}$$

Now normal is

$$y - y_1 = -\frac{(2y_1+1)}{6x_1}(x - x_1)$$

Put $\left(0, \frac{3}{2}\right)$

$$\frac{3}{2} - y_1 = -\frac{(2y_1+1)(-x_1)}{6x_1}$$

$8x_1 = 8y_1x_1$

$\Rightarrow$ $x_1 = 0$ or $y_1 = 1$

Put in equation (1)

at $x_1 = 0$, we get $y_1^2 + y_1 + 10 = 0$

No solution.

at $y_1 = 1$

we get,

$2 - 3x_1^2 + 10 = 0$

$x_1 = \pm 2$

$\therefore$ $\frac{dy}{dx} = \pm 4 = n$

$\therefore$ $|n| = 4$